A Network Model for Bus Routes as Directed Graphs

By Esther Tiey and Low Lik Hern
Jurong Junior College

1. Introduction

In May 2014, the Government of Singapore restructured the public bus industry to "Government Contracting Model". All bus infrastructures such as depots, as well as operating assets are under the ownership of the Government. In May 2015, the Land Transport Authority (LTA) announced that they had awarded the contract to operate to Tower Transit Group Limited. Tower Transit may add to its existing bus services to cater to new transport demand from Jurong East to Clementi. As Jurong Junior College is in the same district as Bulim bus depot, it is fitting to evaluate the bus routes which could be proposed for Tower Transit's consideration by considering its profitability. [1]

2. Weighted Edges Model

2.1 Example

In Figure 2.1, the nodes A to H represent a bus stop each and the edges between these nodes represent a path between the bus stops.

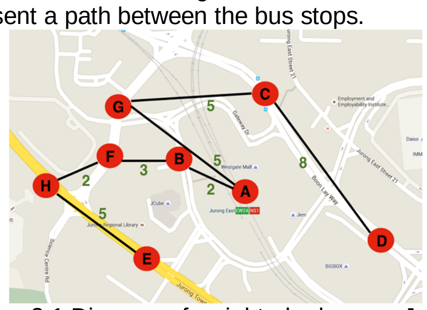

Figure 2.1 Diagram of weighted edges on Jurong East Map

2.2 Assumptions

Assumption 1: Revenue is constant independent of the routes taken.

Assumption 2: The shortest route travelled by the bus is the route with the lowest cost.

To summarize, we are trying to

(max Profit) by (min Operating Cost) by (min Distance Travelled).

2.2 Formulation

Let $V=\{v_i : 1\le i\le n\}$ denote the set of bus stops in Jurong East and Clementi area, where v_1 is the Jurong East interchange (28009) and v_n is the Clementi interchange (17009). This is shown in the ***From*** and ***To*** column in **Figure 2.2**.

Let E be the collection of edges in the network such that $e_{ij}\in E$ if the following two conditions hold:

1. There is a bus that passes through bus stops v_i and v_j travelling in the direction ij,
2. There is no other bus stop between v_i and v_j.

Let W be the collection of weights w_{ij} assigned to each $e_{ij}\in E$ where w_{ij} is the distance between the bus stop v_i and v_j. This is shown in the ***Distance*** column in **Figure 2.2**.

From	To	Distance
28009	28441	0.5
28441	28311	0.4
28311	28631	0.5
28631	28641	0.3
28641	28651	0.6

Figure 2.2: Table showing the distances between any two bus stops

Define also $x_{ij}=1$ if edge (i,j) is on the path considered or 0, otherwise.

In this formulation, we are trying to solve the following linear programming problem. [2]

$$\text{Min} \sum_{edge(i,j)} w_{ij}x_{ij}$$

Such that

$$\sum_{edge(1,j)} x_{1j} - \sum_{edge(i,1)} x_{i1} = 1, \text{ for node } v_1$$

$$\sum_{edge(n,j)} x_{nj} - \sum_{edge(i,n)} x_{in} = -1, \text{ for node } v_n$$

$$\sum_{edge(i,j)} x_{ij} - \sum_{edge(j,k)} x_{jk} = 0, \text{ otherwise.}$$

We wish to select the set of edges with minimal weight, subject to the equality constraint shown above.

2.3 Data Collection

From the discussion in the previous section, to solve the model we will need the information on the set of the nodes, the edges and the weights.

1. From the TransitLink website, we are able to find the distance between any bus stop and the origin, Jurong East bus interchange. With this we can determine the relevant bus stops (nodes). Example is given in Table 2.3.

Distance	Bus Stop Code	Road/ Bus Stop Description
0.0	28009	Jurong East Temp Interchange
0.5	28441	Seventh-Day Advent Ch
0.9	28311	Blk 209

Table 2.3: Distance from Jurong East Interchange to different bus stops

2. The distance (weights) between the bus stops can be obtained by taking the difference between 2 successive entries in the highlighted column in Table 2.4.

	From	To	Distance
0.0	28009	28441	0.5
0.5	28441	28311	0.4
0.9	28311	28631	0.5
1.4	28631	28641	0.3
1.7	28641	28651	0.6

Table 2.4: Raw data example of distance between bus stops.

3. We do this for every bus within the vicinity. Then, we identified the common route taken and removed the repeated bus stop pairs.

2.4 Solution

Implementing the model and solving it using Data Solver in Microsoft Excel, we find the shortest path from Jurong East Bus Interchange to Clementi passes through the following bus stops.

28009-28441-28311-28631-28641-28651-28049-28039-28029-20109-17189-17009

This is depicted in Figure 2.5.

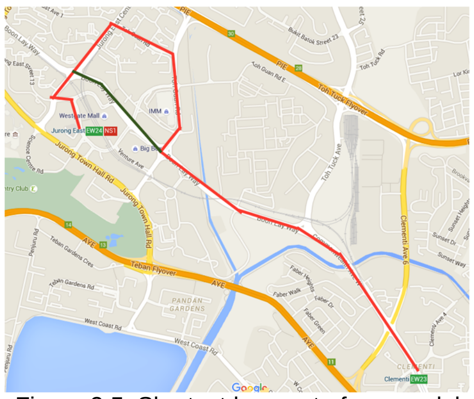

Figure 2.5: Shortest bus route from model

However, if we observe the diagram, the suggested shortest path seems a little strange. The bus will make a round trip before turning into Boon Lay Way, instead of doing it immediately at the nearest junction. It turns out that such route is not possible. The bus stops in between are not directly connected.

In the model, we see that we have neglected the carrying capacity of the buses, which contributes to the profitability of the route. Consequently, in the next section, we propose an alternative model that account for this.

3. Weighted Nodes Model

3.1 Assumptions

Assumption 1: The number of passengers transported at each bus stop is proportional to the number of bus services passing through it.

Assumption 2: Operating cost is the same for all routes.

To summarize, in this model, we are trying to

(max Profit) by (max Revenue) by (max No of passengers) by (max Sum of bus services passing through bus stops).

3.2 Formulation

In this model, the edges in the model described in Section 2 are part of the new nodes. We will also need to introduce 2 dummy nodes to indicate the source and end nodes in this model respectively.

For any two nodes v_1 to v_2 in this model, there is an edge between them if one of the following holds

1. v_1 is the source dummy node and v_2 is an edge in Model 1 that started from Jurong East,
2. v_1 is an edge in Model 1 that terminates at Clementi Interchange and v_2 the end dummy node.
3. The end point of the edge v_1 in Model 1 is the starting point of edge v_2 in Model 1.

In Figure 2.6 , we give an example of a conversion from a weighted nodes model (Left) to weighted edges model (Right).

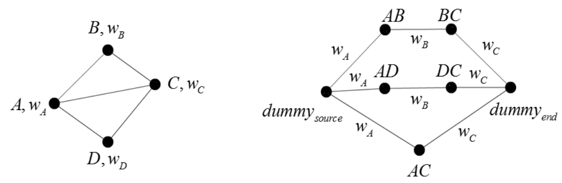

Figure 2.6: Model 1(Left), Model 2(Right)

The weights of the edges in this model are number of bus services that pass through the associated bus stops.

In this formulation, we are trying to solve the following linear programming problem. [3]

$$\text{Max} \sum_{edge(i,j)} w_{ij}x_{ij},$$

Which is equivalent to

$$\text{Min} \sum_{edge(i,j)} \left(-w_{ij}\right)x_{ij},$$

with constraints similar to model in Section 2.

3.3 Data Collection

Using the data from the previous model,

1. We can create incidence matrix and use it to find the set of the new nodes.
2. Note that the entries are negative as we need to multiply the weights with -1. (Table 3.1)

From	To	Weight
A	B	-9
B	C	-8
C	D	-11

Table 3.1: Weights of the e in the new model

3.4 Solution

We implemented the model and solved it using Solver in Microsoft Excel and found the following solution:

28009-28441-28311-28631-28641-28651-28049-28039-28029-20109-17189-17009

We found out that this is exactly the same bus route that we obtained in Model 1.

4. Conclusion

In our analysis of the profitability, we focused on the following 2 factors and tackled them separately.

1. The distance taken by buses. The distance taken by buses should be kept at a minimum.
2. Maximal use of the buses. The buses should pass prime bus stops.

Based on the 2 proposed models, not only the route minimizes the distance travelled, it also maximizes the number of passengers. Consequently, this obtained route is worth a consideration.

The advantage of our models is that the formulation of the problem is simpler and so is the implementation as a linear programming problem in Microsoft Excel.

However, in a more authentic formulation, these 2 factors are interlinked and thus, should be considered together.

Therefore, to improve the model, we need to modify our objective function to include both factors concurrently.

5. References

[1] Transition to a Government Contracting Model for the Public Bus Industry. Web. 21 May 2015

[2] Dreyfus, S. E. "An Appraisal of Some Shortest Path Algorithms." (1967): n. pag. Web. 1 Apr. 2016.

[3] Farahani Reza Zanjirani, Graph Theory for Operations Research and Management: Applications in Industrial Engineering. N.p.: n.p., n.d. Web. 1 Apr. 2016.